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The Twin Paradox and the Conventionality of Simultaneity

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A new approach to understanding the twin paradox, based on the conventionality of simultaneity, is presented and illustrated. The canonical version of the twin paradox is discussed with reference to its historical origins and the standard explanations given for the differential ageing of the twins. It is shown that these are merely specific examples of an infinite class of possible accounts, none of which is privileged. The bounds of this class are given a novel geometrical interpretation. Non-standard versions of the twin paradox are discussed, and the conventionality of simultaneity approach is generalized. (1) The use of general relativity to explain the twins' ageing is also criticized. The application of the conventionality of simultaneity to the twin paradox hopefully provides a way to settle the often discussed issue of the twins' differential ageing.

I. INTRODUCTION

The "twin paradox" has been the subject of a great deal of interest and discussion since the introduction of special relativity by Einstein in 1905. Henri Arzelies has pointed out that while Einstein had suggested the kernel of the paradox it was Paul Langevin in 1911 who first posed the problem in its current more or less standard form.[1] Describing much of the subsequent scholarly discourse concerning the paradox, Arzelies complains that "the same arguments are always advanced, and the same replies given."[2] A review of recent work dealing with the twins in special relativity seems to bear this out. This paper attempts to provide an approach to the twin paradox, suggested by one of us,[3] which will make superfluous much of the standard discussion that Arzelies finds so exasperating.

In Langevin's 1911 paper on space and time he discussed at length the implications of special relativity and presented what came to be called the twin paradox as an "exemple concret," of its implications.[4] Langevin described a scenario in which a traveller leaves the Earth for a distant star at a speed close to the speed of light and returns in the same manner having aged only two years, while on Earth two centuries have elapsed.[5] This is essentially the standard account of the paradox to which is often added the idea that the traveller has a twin who stays on the Earth, so that at the end of the trip the twins will have aged differently.

Langevin did not include his calculations, but the differential ageing can be demonstrated by calculating the proper time, τ, along the two paths through Minkowski space-time as shown in Fig. 1. These are labelled as path 1, from the origin of the Earth's frame to time $t = 2T$ along the vertical axis and path 2, from the same origin to the turning point e and back again, and they correspond to the earthbound and the travelling twin respectively. To obtain proper times one can integrate along each path using the fact that for constant speeds,

$$d\tau = \gamma^{-1}\, dt, \tag{1}$$

where

$$\gamma = 1/(1-v^2/c^2)^{1/2}, \tag{2}$$

t is time measured in the inertial frame of the Earth, v is the speed of departing and returning, and c is the speed of light. Equating this calculation of proper time with the time measured by ideal physical clocks is what has been referred to as the "clock hypothesis."[6] We will assume that this hypothesis holds, although there is some discussion as to when clocks do actually measure proper time.[7] Since γ^{-1} is always less than or equal to 1, the proper time measured along path 2 will always be less than that along path 1. That is to say:

$$\tau_1 = 2T \qquad > \qquad \tau_2 = \gamma^{-1}\,(2T). \tag{3}$$

The differential ageing suggested by Langevin comes directly from the fact that proper time is a path dependant quantity in special relativity.

From these straightforward calculations it is not clear where there is a "paradox" in the story of the twins. One way to make the twin paradox seem paradoxical or at least unexpected would be to note that special relativity predicts reciprocal dilation of moving clocks, according to which each clock appears to move slower than the other, and to contrast this to the non-reciprocal dilation predicted for the round-trip journey.[8] Wesley Salmon refers to this symmetrical time dilation as the "clock paradox" as opposed to the asymmetrical dilation which takes place in the twin paradox.[9] Perhaps confusingly "clock paradox" can also refer to a formulation of the twin paradox which attempts "To avoid ... the [biological] issue of whether a traveller's ageing is in accord with the standard clock that he carries."[10] Because proper times are path dependant quantities the time dilation which produces the "clock paradox" in the first sense fails to produce a truly paradoxical version of the twins' story. With notable exceptions including Herbert Dingle,[11] most commentators agree that the proper times on the two paths of the twin paradox are unambiguously different, and that as such there is formally speaking no paradox. Instead, most of the significant discussion of the twins has focussed on the asymmetry between the two paths and on trying to explain how and when the differential ageing actually occurs. It is these sorts of arguments which we will address.

II. ASYMMETRIES

Langevin himself was the first to emphasize the fundamental lack of symmetry between the path through space-time of his traveller and that of a stationary observer 4 on the Earth.[12] In his 1911 paper he called attention to two asymmetries that have been the basis for many if not most of the standard explanations of the differential ageing advanced since then. The first of these is in the difference between the experience of the traveller and that of the Earth observer if they try to keep track of each other's progress using radio signals.[13] The second fact that Langevin used to support the "dissymetrie" between the two paths was that of the acceleration that the

traveller must undergo in order to return to Earth.[14] Many arguments have been advanced since Langevin's paper up through the 1990s which have followed the lines suggested by these two asymmetries. These arguments can be grouped into those that focus on the effect of different standards of simultaneity in different frames and those that designate the acceleration as the main reason for the differential ageing, many of which also demand the application of general relativity to the problem. delete

The family of explanations of how the twins age differentially based on the relativity of simultaneity effectively includes several different but related approaches. Among these explanations are the radio signal approach first suggested by Langevin and Lord Halsbury's "three brothers" approach. These both explicitly or implicitly remove from consideration the role of the acceleration. Each then tells a story about how during the course of the journey the proper times measured by earthbound and travelling clocks change with respect to one another.

David Bohm gives a detailed version of the radio signal approach. The different experiences he describes of traveller and earthbound observer while maintaining radio contact follows Langevin's qualitative discussion closely. See Fig. 2. From the relativistic Doppler shift equations, Bohm notes that from the point of view of the earthbound observer he or she will receive "first of all a set of slower pulses and later [after time q], another set of faster ones."[15] Where q=T($1+v/c$) is the time that the first signal is received after the travelling twin turns around.

Conversely Bohm concludes that "If the rocket observer were watching the fixed observer he would then see the life of the latter slowed down at first and later speeded up."[16] The change between slow and fast would in this case occur at the time p = T($1-v/c$) when a signal from Earth reaches the traveller at the turnaround point e. See Fig. 3. Bohm concludes that for the travelling twin "the effect of the speeding up more than balanced that of the slowing down. He would not therefore be surprised to find on meeting with his twin that the latter had experienced more of life than he had."[17] Bohm's account of the relative lapse of proper time for each observer does not give the acceleration any special treatment and describes a situation where

each observer sees the other going more slowly than him or herself at first and going faster after a certain moment in time, p or q.

The radio communication solution exemplified by Bohm is similar to the "three brothers" approach suggested by Lord Halsbury. This is a situation where instead of turning the corner at e the travelling twin's clock is synchronized with a third clock carried by a third sibling already moving at the opposite velocity towards the Earth; the time measure by both clocks will together give us the proper time along the whole of path 2.[18] This is intended to remove any question of the effect of acceleration on the motion.[19] The difference in measurements of proper times on the two paths, according to those who have adopted this approach is (as in Bohm's discussion) based on the relativity of simultaneity. Each inertial frame, stationary, departing and returning, has lines of simultaneity, horizontal, and parallel to the lines re and se respectively defined by the Einstein convention, as shown in Fig. 4. Therefore on outgoing and returning legs both travelling and stationary clocks seem to be going faster than each other, but the change of inertial frames at e constitutes a change of lines of simultaneity which results in a jump ahead between the times r and s as measured on the moving clocks with respect to the stationary clocks. The "missing time" between r and s becomes then the reason for the differential ageing.

In addition to those who have used simultaneity considerations to account for the twins age difference, some have taken Langevin's second asymmetry, the role of the direction-reversing acceleration, as essential to a complete explanation of the paradox. Many seem to feel that the introduction of general relativity and a gravitational field at the point of acceleration is the best way to explain this second
8
asymmetry. Bohm also expresses this view and notes that "two clocks running at places of different gravitational potential will have different rates."[20] In a recent article in this journal, S.P. Boughn uses just this observation to interpret his explanation of the differential ageing in a version of the twin paradox.[21] In his paper Boughn argues that "identically accelerated twins," a scenario treated more fully in Sec. VI, also age differently and that this ~~is the~~ effect is due to the way that proper

delete time is calculated in a uniform gravitational field.[22] Although ~~he uses only special~~ relativity to calculate ~~his result,~~ Boughn is one of the most recent ~~proponents of an~~ understanding of the twin paradox based on general relativity. delete

III. THE CONVENTIONALITY OF SIMULTANEITY

11 The result of both simultaneity based and general relativity motivated explanations of the twin paradox has been a situation in which discussion centers on trying to say where and when the traveller loses time against the Earth. However, there is another approach to the differential ageing problem that promises to lay all of this discussion aside. One of us has recently suggested the application of the conventionality of simultaneity, introduced by Reichenbach and Grünbaum, to the twins problem.[23] 12 This approach to simultaneity denies that there is a fact to the matter in designating one standard of simultaneity within the bounds of the light cone; applied to the twins, this undermines much of the discussion of their specific relative ageing.

As philosopher Michael Friedman puts it the conventionality of simultaneity implies that only proper time has "objective status in special relativity."[24] More 13 specifically this approach refers to Reichenbach and Grünbaum's concept of "topological simultaneity."[25] This is simply the assertion that in using light signals to synchronize two spatially separated clocks, at points a and b shown in Fig. 5, one need not divide the difference of transmission, t_1, and reception, t_3, of a signal by two as originally described by Einstein. Doing so gives Einstein's convention of simultaneity, represented by t_{2E}, which is equivalent to assuming the isotropy of the one-way speed of light. Instead, one could choose any time, t_2, measured at position a between t_1 and t_3 to be simultaneous with the time of reception of the signal at position b. Another way of saying this is that the interval from t_1 to t_3 is topologically simultaneous with the time of reception recorded at b. John Winnie has investigated some consequences of this approach to establishing simultaneity, using Reichenbach's notation:[26]

$$t_2 = t_1 + \varepsilon(t_3 - t_1), \tag{4}$$

such that $0 < \varepsilon < 1$. When $\varepsilon = 1/2$, this is equivalent to the Einstein convention. This process of synchronizing spatially separated clocks can also be done in moving frames. If the Einstein convention is used from the point of view of the moving frame one will get, from the point of view of the rest frame, the shifted lines of simultaneity which are so important to the Halsbury "three brothers" explanation of the differential ageing. Although he does not explicitly mention the twins, Winnie points out that any simultaneity criterion, such that $0< \varepsilon <1$, may in fact be applied without affecting the differential ageing in what is equivalent to a Halsbury type phrasing of the twin paradox.[27] This is what one would expect if the choice of ε is truly one of convention. However, Winnie also concludes that the standard time dilation in special relativity described by the phrase "moving clocks run slow" is in some ways an artifact of the Einstein convention. Winnie calculates specific criteria, i.e. values of ε, according to which clocks can be seen to run synchronously in rest and moving frames.[28] To do this and remove any one-way time dilation, he shows that one must choose different values of ε for when the clocks being synchronized are receding, ε_r, or approaching, ε_a, with respect to the rest frame, and that these values are additive inverses of each other, such that:[29]

$$\varepsilon_r + \varepsilon_a = 1. \tag{5}$$

What Winnie's work suggests for the twin paradox is that while round-trip differential ageing is not dependent on convention, the one-way description of relative clock rates is. This is exactly the position the authors support, that any choice of simultaneity criterion, ε, will give the overall difference in age for the twins, but that each different choice will represent an equally acceptable story about the relative rates of clocks along each portion of the journey. If this is the case then any of the discussions of where or when during his or her journey the traveller gains on the earthbound twin become equally conventional and thus entirely uninteresting.

IV. CONVENTIONALITY OF SIMULTANEITY AND THE TWINS' AGEING

In order to apply the conventionality of simultaneity to the problem of relative rates of clocks in the twin paradox, we consider a situation in which the travelling twin continuously sends and receives signals from the Earth and uses these to set upper, *u*, and lower, *l*, bounds on possible simultaneity assignments for his or her clock. See Fig. 6.[30] We subsequently plot the proper time along each path against one another which produces a parallelogram, as shown in Fig. 7, the upper and lower boundaries of which are the bounds on possible times measured by the traveller for a given instant on the earthbound clock. That is to say that for each instant of proper time measured along path 1, τ_1, there exists a range of proper time values along path 2, τ_2, which would all be equally good choices to be considered simultaneous with that particular value of τ_1. It is important to note that the diagonal of the parallelogram lies below the line of slope equals 1 such that the differential ageing by the end of the journey is undisputed. This parallelogram, OPQR, also allows one to see that it would be possible for either clock to run more quickly than the other over any particular interval within its bounds. The essence of the conventionality of simultaneity approach to the twin paradox can be made apparent by remarking that any non-decreasing curve inside the parallelogram would be equally acceptable as a way of describing the relative rates of the two clocks.

With this approach to the twin paradox it is easy to see that any discussion about where during the journey the differential ageing takes place is unnecessary. In fact, many of the standard explanations can be plotted onto the parallelogram. Two of those already discussed involve the use of the Einstein convention from the point of view of the traveller.[31] In the first, one could simply half the difference between the traveller's sending and receiving times over the whole journey to establish the progress of the twins' clocks relative to one another. Graphically this would mean taking the average between the upper and lower boundaries of the parallelogram, as depicted by the dashed line segments in Fig. 7. Adopting this method, the traveller's clock seems to run more quickly than the Earth's up to $\tau_1 = T(1-v/c)$, the time in the Earth's frame

that the first signal reaches the turnaround point. Then the traveller's clock seems to lose ground against the Earth's until $\tau_1 = T(1+v/c)$, the time that the Earth receives its first signal after the turnaround. Thereafter the traveller again ages more quickly but the overall effect is such that his or her total age is less than that recorded on Earth.

The Halsbury "three brothers" approach to explaining of the differential ageing,, can also be represented as a curve inside the parallelogram, shown by the dashed line segments in Fig. 8. To get this curve, the Einstein convention for simultaneity for a frame receding with velocity, v, is used until the traveller reaches his or her halfway point, half of τ_2, implying that the traveller is ageing more quickly. On the second half of τ_2, the same convention for an approaching frame is used, and the traveller ages more quickly again. The overall youth of the traveller is due then to the "missing time" from the traveller's journey which is represented by the horizontal section in which the Earth ages instantaneously from his or her point of view. Conversely, from the Earth, the travelling twin's clock seems to stand still during this period.

Infinitely many other stories may also be told which fit into the bounds of convention set by the parallelogram. As we have seen, the simultaneity criterion, ε, can be chosen so as to eliminate one-way time dilation if $\varepsilon = \varepsilon_r$, for receding clocks, or ε_a, for approaching ones. The result of choosing these criteria is represented by the dashed line segments in Fig. 9. During the first half of the travelling twin's journey his or her clock runs in synchrony with the clock on earth, the dashed line segment runs along the line of slope one. These lines are also parallel during the second half of the twin's journey where the clocks run at the same rates again. The overall differential ageing is caused by the horizontal dashed segment over which, from the Earth, the traveller's clock stands still. Interestingly, the additive inverse relationship of Eq. (5) can be seen readily on the parallelogram. Modifying Reichenbach's notation in Eq. (4):

$$\varepsilon = (t_2 - t_1)/(t_3 - t_1). \qquad (6)$$

apostrophe While from the travellers point of view, t_3 is equal to the upper boundary of the parallelogram, t_1 the lower boundary, and t_2 the chosen simultaneous moment

represented by the dashed line segments. This implies that on the first half of the traveller's journey,

$$\varepsilon_r = B/(A+B), \tag{7}$$

where A and B are the magnitudes labelled on Fig. 9. On the second half of the journey,

$$\varepsilon_a = A/(A+B). \tag{8}$$

This gives the result that:

$$\varepsilon_r + \varepsilon_a = A+B/(A+B) = 1, \tag{9}$$

as expected.

The boundaries of the parallelogram can also be seen to represent the approach to explaining the twin paradox, exemplified above in Bohm's discussion, which uses Doppler shifted radio signals. Looking more closely at the boundaries of the parallelogram we can see that the lower boundary has the slope $[(1-v/c)/(1+v/c)]^{1/2}$ and the upper boundary has the slope $[(1+v/c)/(1-v/c)]^{1/2}$ where v is taken to be the outgoing velocity of the traveller and $-v$ the returning velocity. This is not surprising as these slopes are the relative rates of the measurement of proper times in frames moving with respect to one another. This relationship can be seen in the relativistic Doppler shift equation according to which:

$$\tau' = [(1+v/c)/(1-v/c)]^{1/2}\, \tau, \tag{10}$$

where τ' is the period of radiation received in a frame moving with velocity v and τ is the period of the radiation in the rest frame, in the situation where the radiation is propagating in the same direction as v. Noting that $\tau_2 = \Sigma\, \tau'$ and $\tau_1 = \Sigma\, \tau$ over their respective paths, and that the Doppler shift equation describes the periods of radiation on the upper bound and the multiplicative inverse describes the lower bound, the slopes of the sides of the parallelogram can be easily confirmed.

Looking to the story Bohm tells of the twins relative progress we can see that he is actually describing the two boundaries of the parallelogram which come directly from the Doppler shifted signals he sets out to discuss. Bohm first discusses the appearance of signals coming from the traveller as seen on the Earth. Looking at the

parallelogram, we can see Bohm's explanation[32] by looking at the lower boundary ORQ in Fig. 9. Taking this approach, the Earth observer sees the travelling twin ageing more slowly up to the time $p = T(1+v/c)$, represented by segment OR of the parallelogram. Subsequently he or she sees the traveller ageing more quickly than earthbound clocks, segment RQ.

From the other point of view, Bohm expects that the moving twin will see the Earth's clock running slower than the moving clock up until the time $q = T(1-v/c)$, and subsequently he or she will see the Earth's clock running more quickly than the moving clock.[33] This is exactly the story that is represented by the upper boundary, OPQ, of the parallelogram in Fig. 9. Bohm explains the differential ageing by pointing out that the speeding up of the Earth's clock witnessed by the traveller after time q "more than balanced" the slower relative rate prior to q.[34] The use of the parallelogram makes it obvious that the Doppler shifted signal approach to the paradox is concerned with the outer bounds of an infinite number of acceptable stories about the twins relative rates of ageing.

V. THE TWINS ON NON-STANDARD PATHS, THE CONVENTIONALITY APPROACH GENERALIZED

Another approach to explaining the differential ageing of the twins without reference to the point of acceleration has been to put the two paths onto cylindrical 9) space-time. In this way the stationary twin is considered to travel up the cylinder, the time axis parallel to the axis of rotation of the cylinder, and the travelling twin to depart and return by simply going around the cylinder at a constant velocity. The calculation of the proper times on the cylinder has been done recently by more than one individual.[35] At first it might seem that one might be able to get a real paradox out of this situation without the obvious asymmetry in the two paths. That is the asymmetry provided by the acceleration and change in direction on the travellers path. This turns out not to be possible because of the structure of simultaneity relations in

cylindrical space-time.[36] This should not be surprising as some asymmetry must exist between the two paths in order to calculate different proper times. ← 20

One place to observe the requirement of an asymmetry to get differential ageing is in the Hafele, Keating experiment. In this experiment, differential aging was observed on two atomic clocks travelling on jets at the same speed around the Earth in opposite directions.[37] The rotation of the Earth provided the asymmetry that was necessary to produce the difference in proper times. These two paths without the rotation of the Earth can be compared schematically to the two paths going around in different directions in cylindrical space-time. The addition of the rotation gives us a scenario which looks roughly like the twin paradox in cylindrical space-time.

Using flat non-cylindrical space-time we can set up an idealized one-dimensional Hafele, Keating situation with two symmetrical paths for each twin corresponding to the case when the Earth's rotation is not considered. See Fig. 10. It turns out that using the Einstein convention of simultaneity implies a specific story about the relative rates of clocks even when there is no overall differential ageing. Taking the same approach as above, we can consider that the twin on path 2 checks the progress of the other by sending and receiving signals. With this information the twin on path 2 again sets upper and lower bounds for acceptable values of its own proper time, τ_2, for each given value of the other twin's proper time, τ_1. The result this time is a symmetrical hexagon with its diagonal along the line of slope 1, so that there is no overall differential ageing. See Fig. 11. The slopes of the sides of the hexagon can once again be explained using the Doppler shift equation, Eq. (10), according to which, as previously stated, the lower boundary has the slope $[(1-v_r/c)/(1+v_r/c)]^{1/2}$ and the upper boundary has the slope $[(1+v_r/c)/(1-v_r/c)]^{1/2}$, where this time v_r is the relative velocity between the twins each moving with velocity v. Using relativistic velocity addition:

$$v_r=2v/(1+v^2/c^2), \tag{11}$$

from $\tau_1= 0$ to p, where $p=[T-T(2v/c)/(1+v/c)]\gamma^{-1}$, which corresponds to segments OP and OT in Fig. 11. Plugging into the Doppler shift equation gives slopes for these

segments of $(1+v/c)/(1-v/c)$ and $(1-v/c)/(1+v/c)$ respectively. The relative velocity, v_r, is zero from $\tau_1 = p$ to q, where $q=[T+T(2v/c)/(1+v/c)]\gamma^{-1}$, which implies that segments PQ and TS are both of slope 1. The other boundaries are similarly calculated and should be apparent from symmetry.

The dashed line segments inside the hexagon represent the story given if the Einstein convention is used in the sense that the average of the upper and lower bounds is taken everywhere. As we can see, use of this convention implies that the clocks on different paths are seen to move faster or slower than each other at different moments during the journey even when there is no overall differential ageing. In fact no single simultaneity criterion, ε, will pick out the diagonal which seems to make the most sense as a description of the clocks in this situation. The arbitrary nature of the implications of any single criterion supports the conventionality approach of setting the boundaries and not discussing paths within them.

We can also create a one-dimensional Hafele, Keating experiment in which there is differential ageing by adding an additional velocity along one path. See Fig. 12. In this situation the magnitude of the velocities on path 1 is less than that of the velocities on path 2, and the overall ageing of the twin on path 1 will be greater. A hexagon can also be drawn to incorporate the possible values of one proper time versus the other, as shown in Fig. 13. The slopes of the sides, as in the previous hexagon, can be given using the relativistic Doppler shift equation, Eq. (10) and the relative velocities of the two twins using relativistic addition of velocities. As one would expect, substitution of the same velocity for paths 1 and 2 gives us back the symmetric hexagon, and substitution of zero velocity for one of the paths gives us the parallelogram from the standard twin paradox.

Some general features of this approach to depicting the relative progress of clocks between two paths in Minkowski space-time can be observed. First of all, it is possible to construct a region of possibly simultaneous points for any two paths. The Doppler shift equation relating periods of signals can be used to sum over all periods to get the upper and lower bounds on this region as long as relative velocity along each

path is constant over each individual period. Much simpler methods can be used to calculate these bounds if the paths are straight and accelerations are instantaneous. In this situation, one can see from the examples done so far that the number of vertices, V, on the boundary of the simultaneity region is given by,

$$V = 2(n+1), \tag{12}$$

where n is the number of instantaneous points of acceleration on the twins' paths excluding the accelerations at separation and return.

The conventionality of simultaneity approach to the twin paradox also clarifies some implications of a method for estimating distance suggested by Clive Kilmister. Kilmister has suggested that the traveller could keep track of his or her distance from the origin of the rest frame using the same signals used above to discuss simultaneity.[38] By this method, described as a "radar" method by Hermann Bondi, the travelling twin could estimate distance from the Earth by estimating the time it takes for a signal to make the trip using the Einstein convention and multiplying by the speed of light.[39] This is equivalent to saying that the distance, d, for a specific value of τ_1, the proper time measured on path 1, is given by:

$$d = c(t_u - t_l)/2, \tag{13}$$

where t_u and t_l are the upper and lower bounds on the value of proper time on path 2, for that value of τ_1. From the parallelogram and hexagons we have already constructed we can get an idea of how the quantity $(t_u - t_l)/2$ varies at different proper times τ_1. Multiplying by c, we can get the distance estimate of Eq. (13). For the standard twin paradox situation the radar method implies that the distance between twins is constant at a maximum distance, d_m, near the change of direction. See Fig. 14. In the symmetrical one-dimensional Hafele, Keating situation, the radar method also gives an artificially low estimate of distance which implies that the relative velocity between the twins is lessened near the turning points. The distortion of these distance estimates is a result of the use of the radar method, and the specific simultaneity criterion it assumes, and these strange results are artifacts of its adoption. This

example demonstrates, this time from the point of view of relative distance instead of relative ageing, the arbitrary results of choosing a single simultaneity criterion.

Role of Acceleration

VI. THE ~~USE OF GENERAL RELATIVITY~~ CRITICIZED

Finally the conventionality approach to the twins differential ageing can also be used to show that discussions which try to pin the age difference to the direction-reversing acceleration are missing the point. The most recent such discussion was that between S.P. Boughn and co-authors Edward A. Desloge and R.J. Philpott in the pages of this journal.[40] Roughly speaking this interchange was inspired by the idea that two twins which undergo the same acceleration ~~will~~ Eventually age differentially as a result. (21) This was cited by Boughn as providing an "important insight into the behavior of clocks in a uniform gravitational field," with a mind to ~~the~~ application ~~of general relativity~~ to the point of acceleration.[41] Desloge and Philpott responded by describing (22) in more detail the paths on Minkowski space-time ~~that this~~ Boughn's scenario requires.[42] A (23) version similar to that which they describe is pictured in Fig. 15. In this case the twins are separated symmetrically, given the same acceleration into a new frame at a point *e* in time, and brought back together symmetrically with respect to their new frame. At the end, the twin on path 2 has a greater total elapsed proper time.

Using the conventionality of simultaneity approach on this version of the twin paradox, one could in principle draw a region of possible simultaneous points that would have fourteen vertices according to our previous general observations, see Eq. (12). Any path within this region would be an acceptable account of the differential ageing. One need not construct the entire region to see that assigning the difference in age to the point of acceleration is only one of these accounts. We can see this in a rough diagram of the shape of this region around the point of acceleration. See Fig. 16. The dashed line segments designate a story that allows the differential ageing to take place at the point of acceleration. However, it is obvious that many other non-decreasing curves could fit within the appropriate bounds. (24)

VII. CONCLUSIONS

Having discussed a new approach to looking at the twin paradox with regard to where and when the differential ageing occurs, it is possible to conclude that the conventionality of simultaneity and in particular the concept of topological simultaneity provides a means to put an end to this question. One can conclude that any explanation of relative ageing that stays within the bounds set by the light cone is equally valid. In addition, discussion based on the application of particular simultaneity criteria, i.e. ε values, either to establish simultaneity or estimate distance is also uninteresting as it discusses only one of an infinite number of conventional ways to approach the problem. Perhaps the method discussed in this paper, the conventionality of simultaneity applied to depicting the relative progress of two travellers in Minkowski space-time, will settle the issue of the twin paradox, one which has been almost continuously discussed since Paul Langevin's 1911 paper.

[1]Henri Arzelius, *Relativistic Kinematics*, (Pergammon Press, Oxford, 1966), p. 187.

[2]Ibid., p. 189.

[3]Michael Redhead, "The Conventionality of Simultaneity," in *Problems of Internal and External Worlds: Essays in Honor of Adolf Grunbaum*, J. Earman et. al. eds., (University of Pittsburgh Press, Pittsburgh, 1993), pp. 103-128.

2) [4]Paul Langevin, "L'Évolution de l'Espace et du Temps," Scientia **10**, 49 (1911).

(3) [5]Ibid., 50.

[6]L. Marder, *Time and the Space Traveller*, (University of Pennsylvania Press, Philadelphia, 1971), p. 91.

[7]For a recent treatment of the clock hypothesis see Clive Kilmister and Barrie Tonkinson, "Pragmatic Circles in Relativistic Time Keeping," in *Correspondence,*

Invariance and Heuristics: Essays in Honor of Heinz Post, Steven French and Harmke Kamminga eds., (Kluwer Academic, Dordrecht, 1993), pp. 207-225.

[8]For a discussion of this attempt to produce a paradox see W.H. Newton-Smith, *The Structure of Time*, (Routledge and Kegan, London, 1980), pp. 187-195.

[9]Wesley C. Salmon, *Space, Time, and Motion: A Philosophical Introduction*, (Dickenson, Encino, 1975), p. 95.

[10]Marder, p. 73.

[11]For a detailed discussion of Herbert Dingle's work on the twin paradox see Hasok Chang, "A Misunderstood Rebellion: The Twin Paradox Controversy and Herbert Dingle's Vision of Science," Studies in History and Philosophy of Science **24**, 741-790 (1993).

[12]Langevin, 51-52.

[13]Langevin, 50-52.

[14]Langevin, 52.

[15]David Bohm, *The Special Theory of Relativity*, (W.A. Benjamin, New York, 1965), pp. 168-170.

[16]Ibid., p. 171.

[17]Ibid., p. 171.

[18]For discussion of this version of the paradox see Salmon, pp. 96-97;H. Bondi, "The Spacetraveller's Youth," Discovery **18**, 505-510 (1957).

[19]Salmon, p. 97.

[20]Bohm, p. 166.

[21]S.P. Boughn, "The case of the identically accelerated twins," Am. J. Phys. **57**, 793 (1989).

[22]Ibid., 791-3.

[23]Redhead, loc. cit.

[24]Michael Friedman, *Foundations of Space-Time Theories*, (Princeton University Press, Princeton, 1983), p. 166.

[25]Redhead, loc. cit.

[26]John A. Winnie, "Special Relativity Without One-Way Velocity Assumptions: Part I," Philosophy of Science **37**, ~~82-83~~ 81-97 (1970).

[27]Ibid., 96-97.

[28]Ibid., 88-91.

[29]Ibid., 88-9.

[30]The initial stages of the analysis here follow that included in Redhead, loc. cit.

[31]Both of these examples were also part of the analysis in Redhead, loc. cit.

[32]Bohm, pp. 168-169.

[33]Bohm, pp. 170-171.

[34]Bohm, p. 171.

[35]Tevian Dray, "The twin paradox revisited," Am. J. Phys. **58**, 822-825 (1990); R.J. Low, "An acceleration-free version of the clock paradox," Eur. J. Phys. **11**, 25-27 (1990).

[36]Ibid.

[37]J.C. Hafele and Richard E. Keating, "Around the world atomic clocks: Predicted relativistic time gains," Science **177**, 166-168 (1972); J.C. Hafele and Richard E. Keating, "Around the world atomic clocks: Observed relativistic time gains," Science **177**, 168-170 (1972).

[38]Kilmister et. al., p. 214.

[39]Hermann Bondi, *Relativity and Common Sense: A New Approach to Einstein*, (Heinemann, London, 1965), pp. 34-35.

[40]Boughn, ~~791-3~~ loc. cit; Edward A. Desloge and R.J. Philpott, "Comment on 'The case of the identically accelerated twins,' by S.P. Boughn," Am. J. Phys. **59**, 280-281 (1991).

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[41]~~Boughn, 793.~~

[42]~~Desloge et. al.~~, 281.

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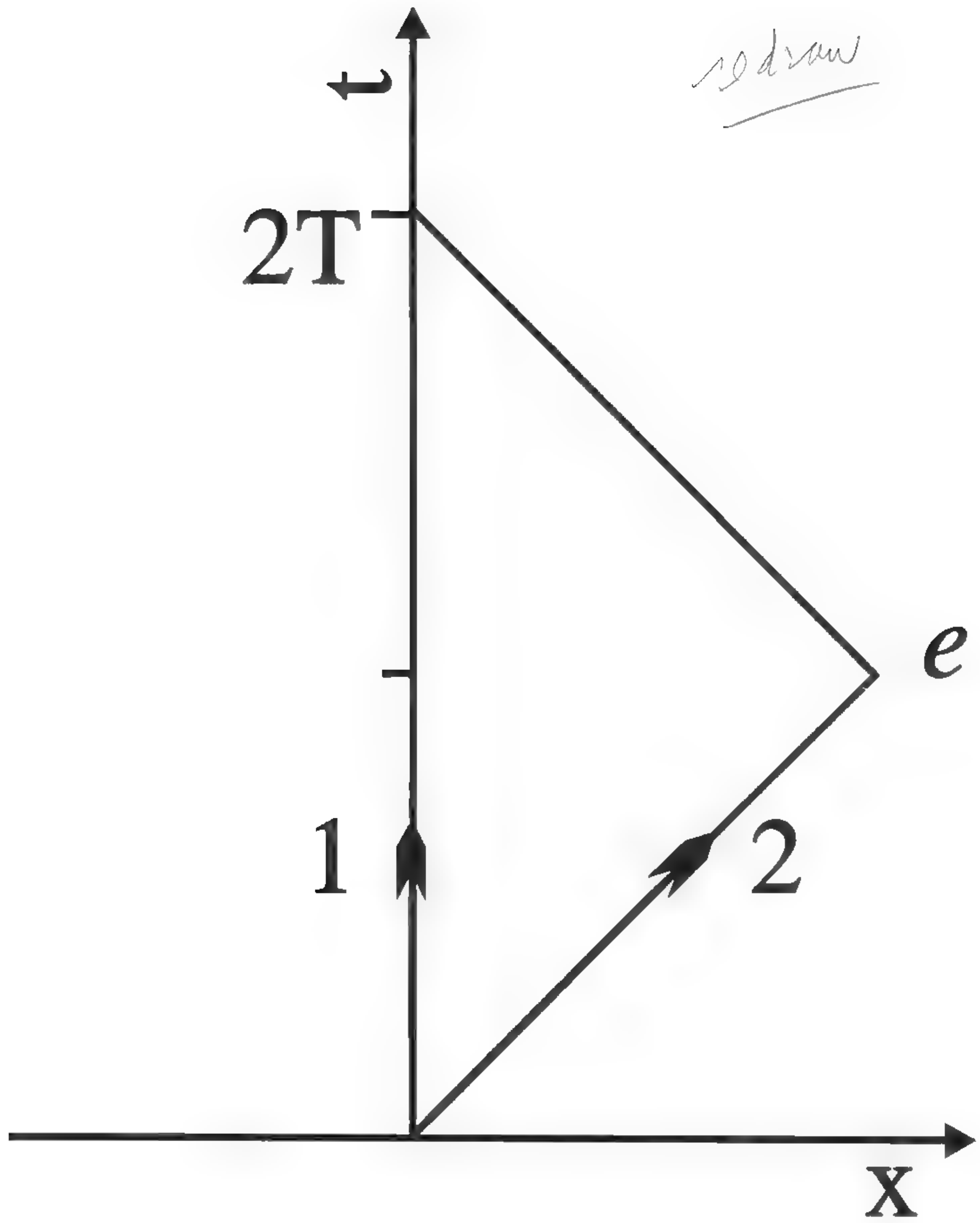

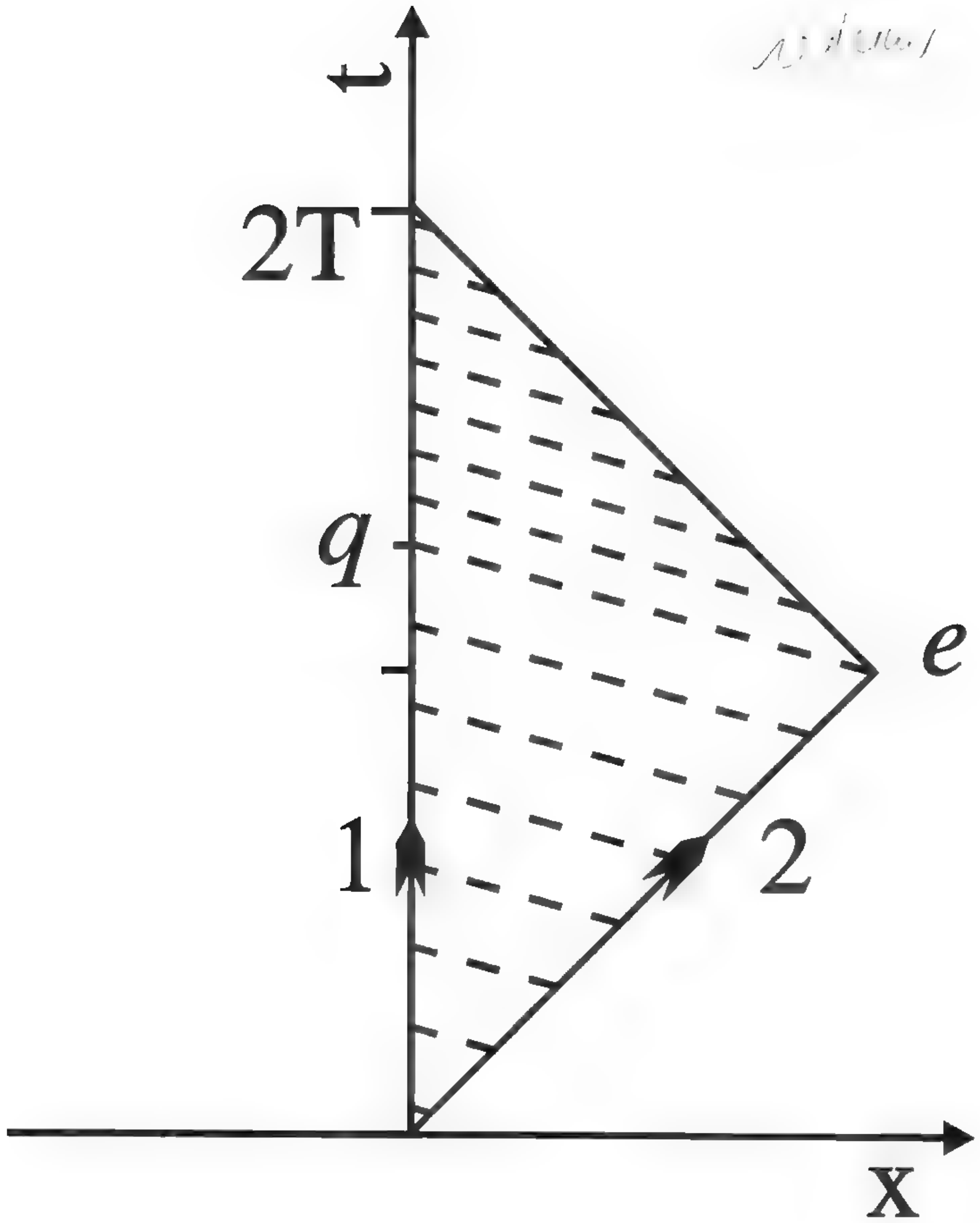
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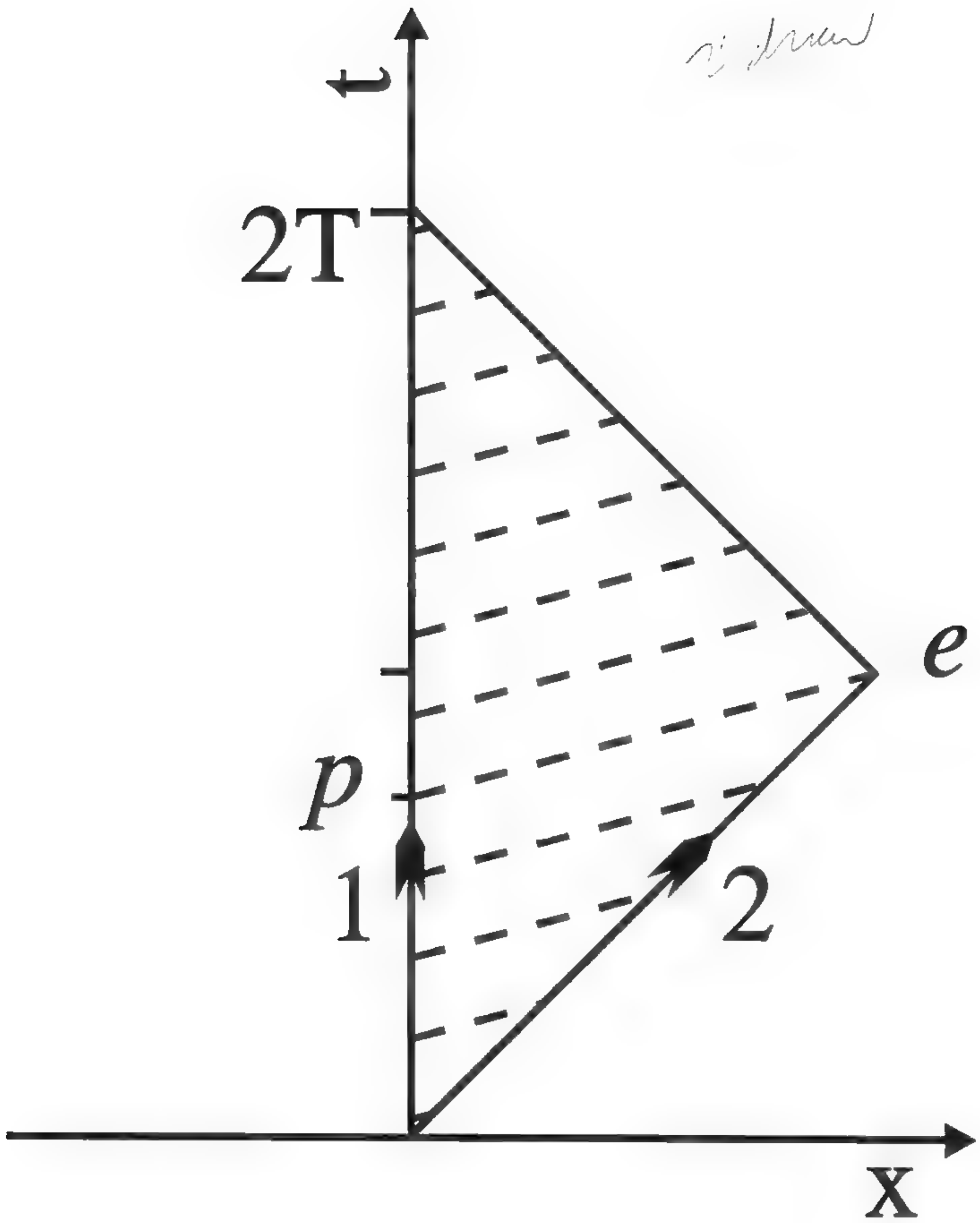
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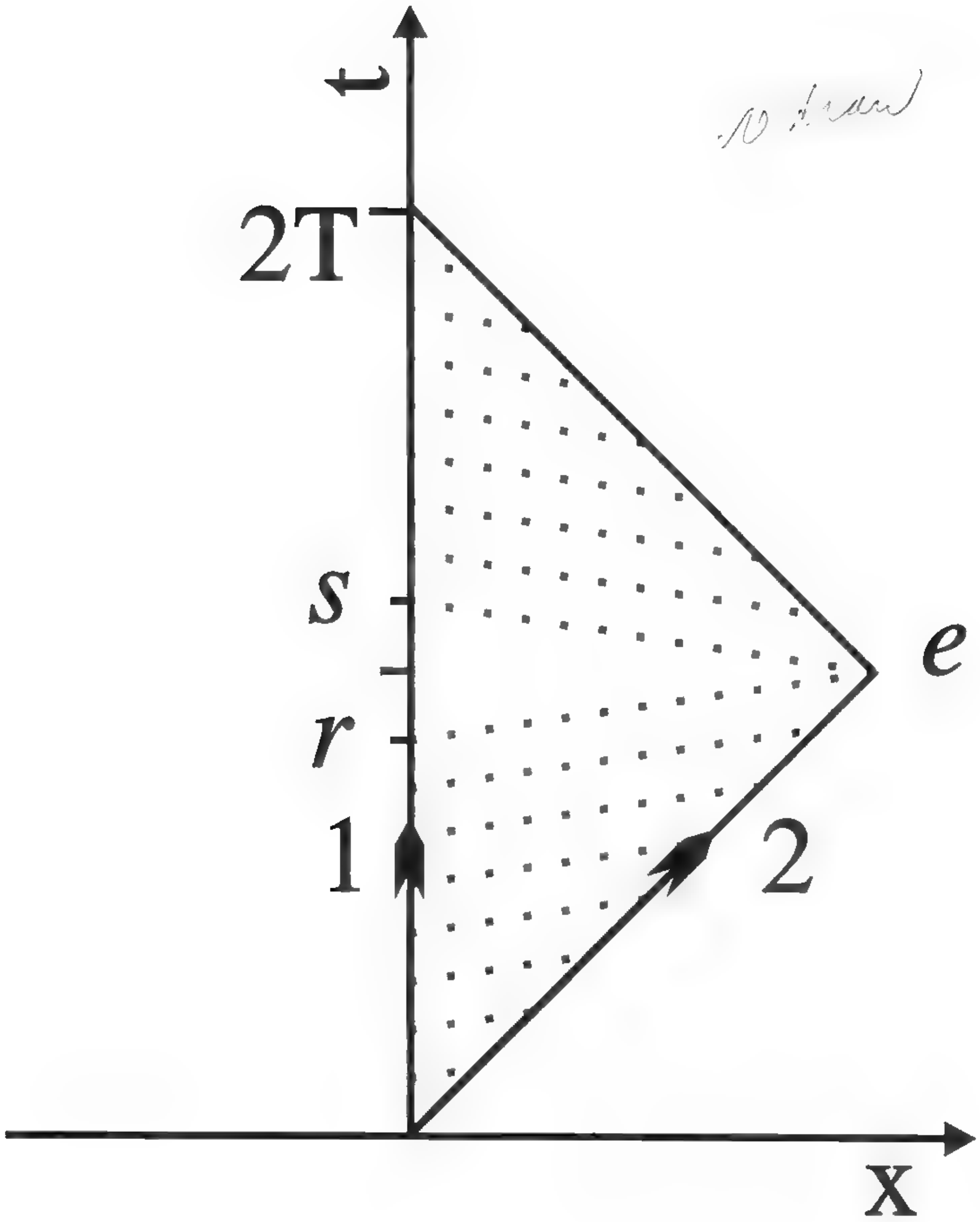
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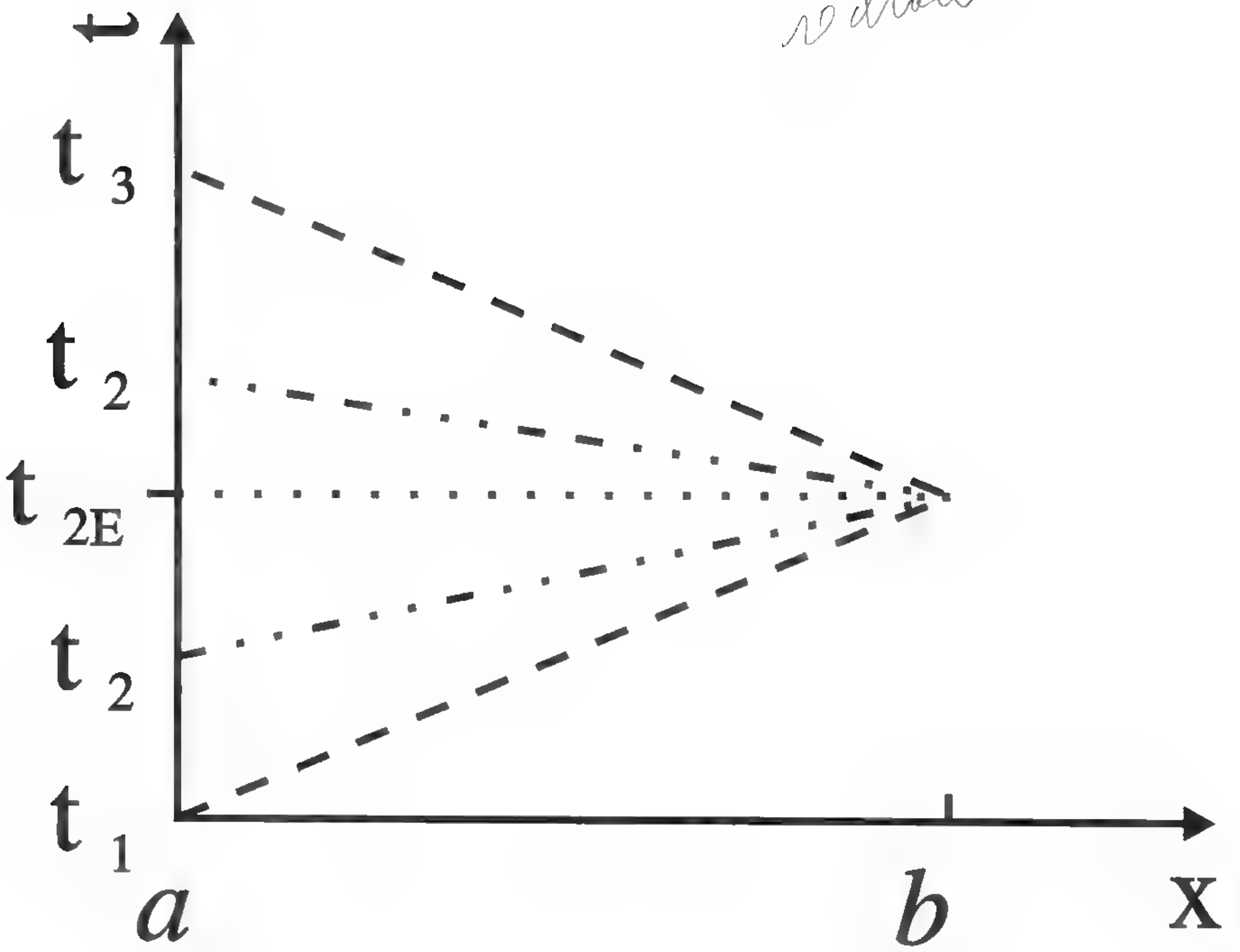

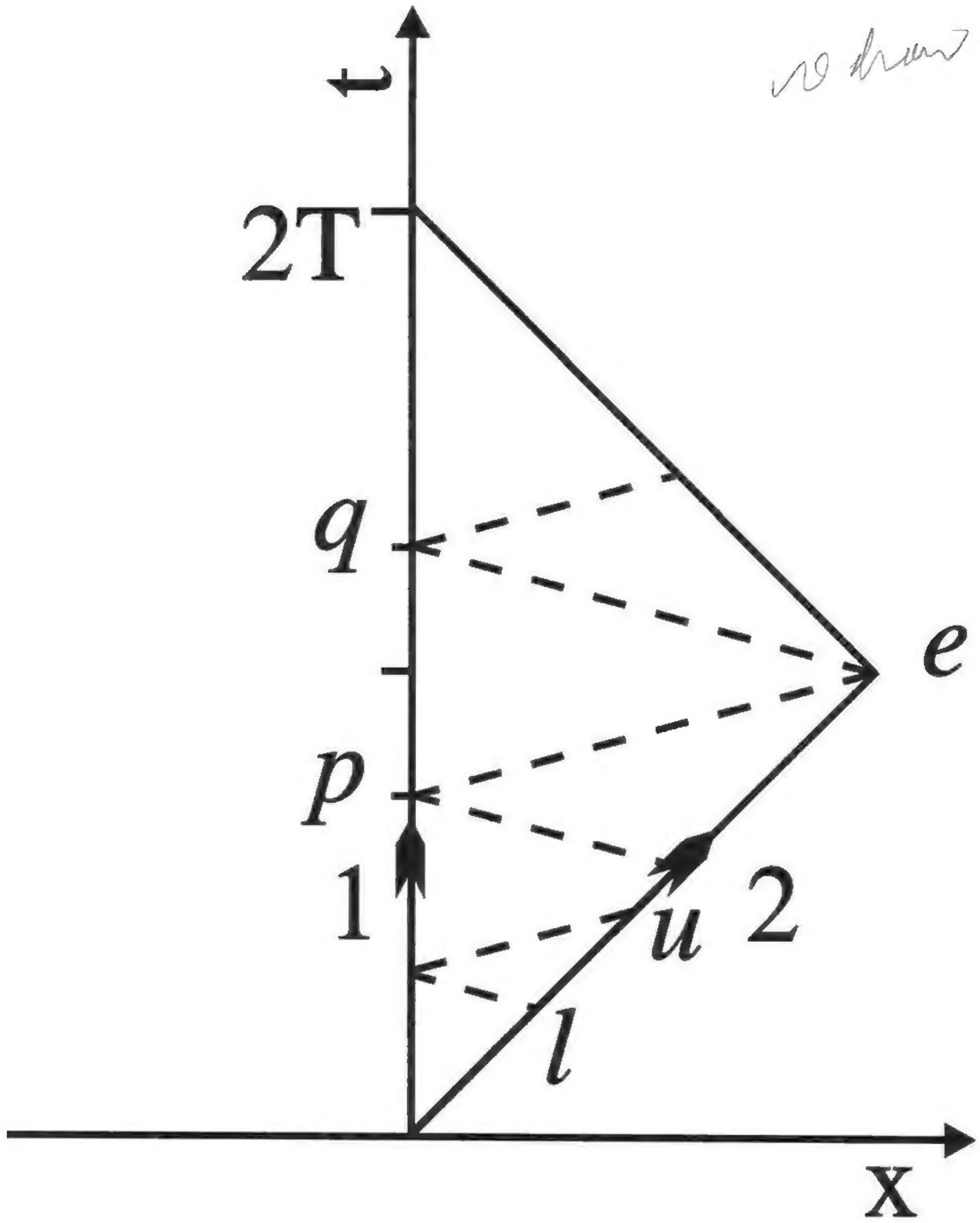

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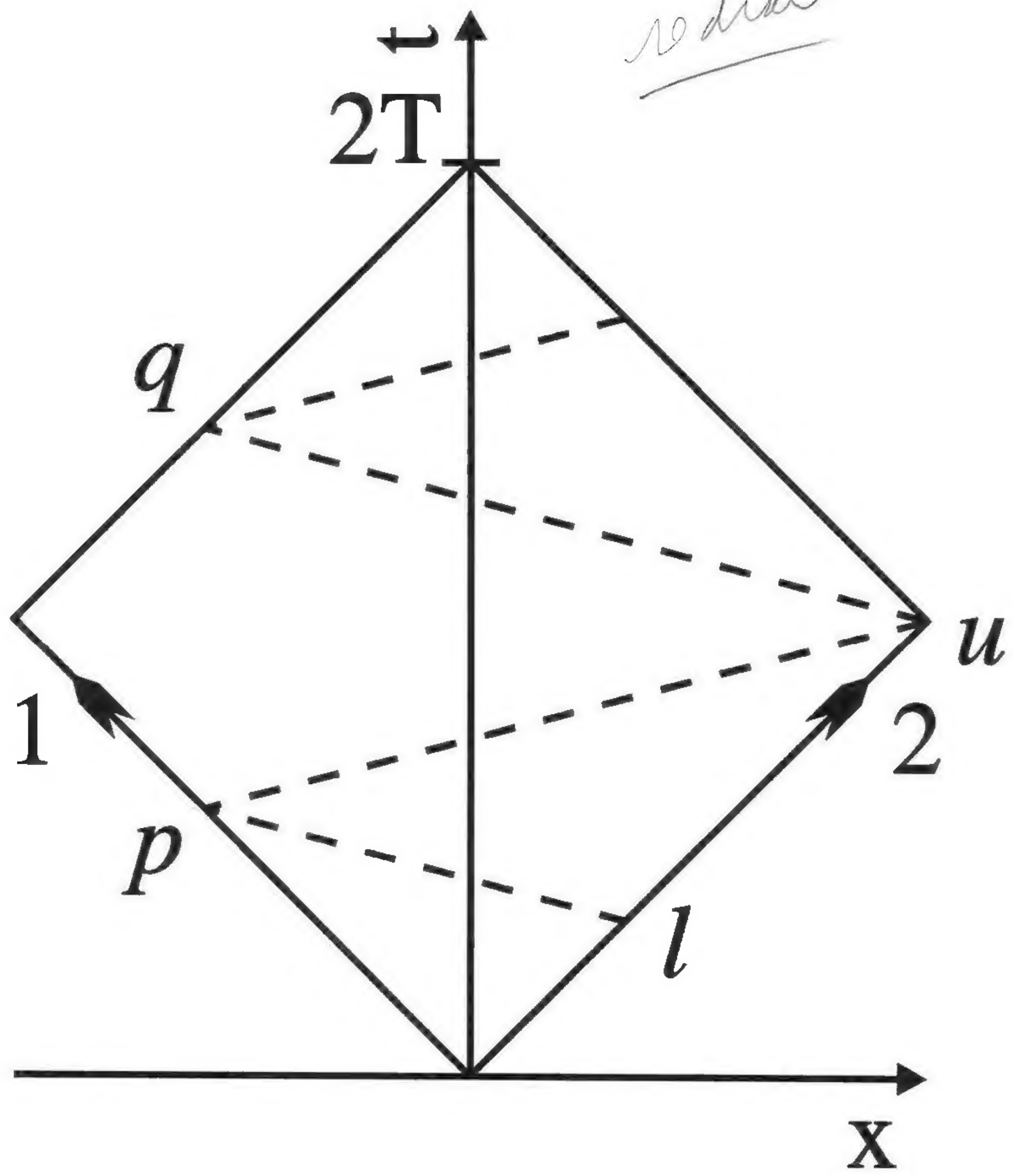

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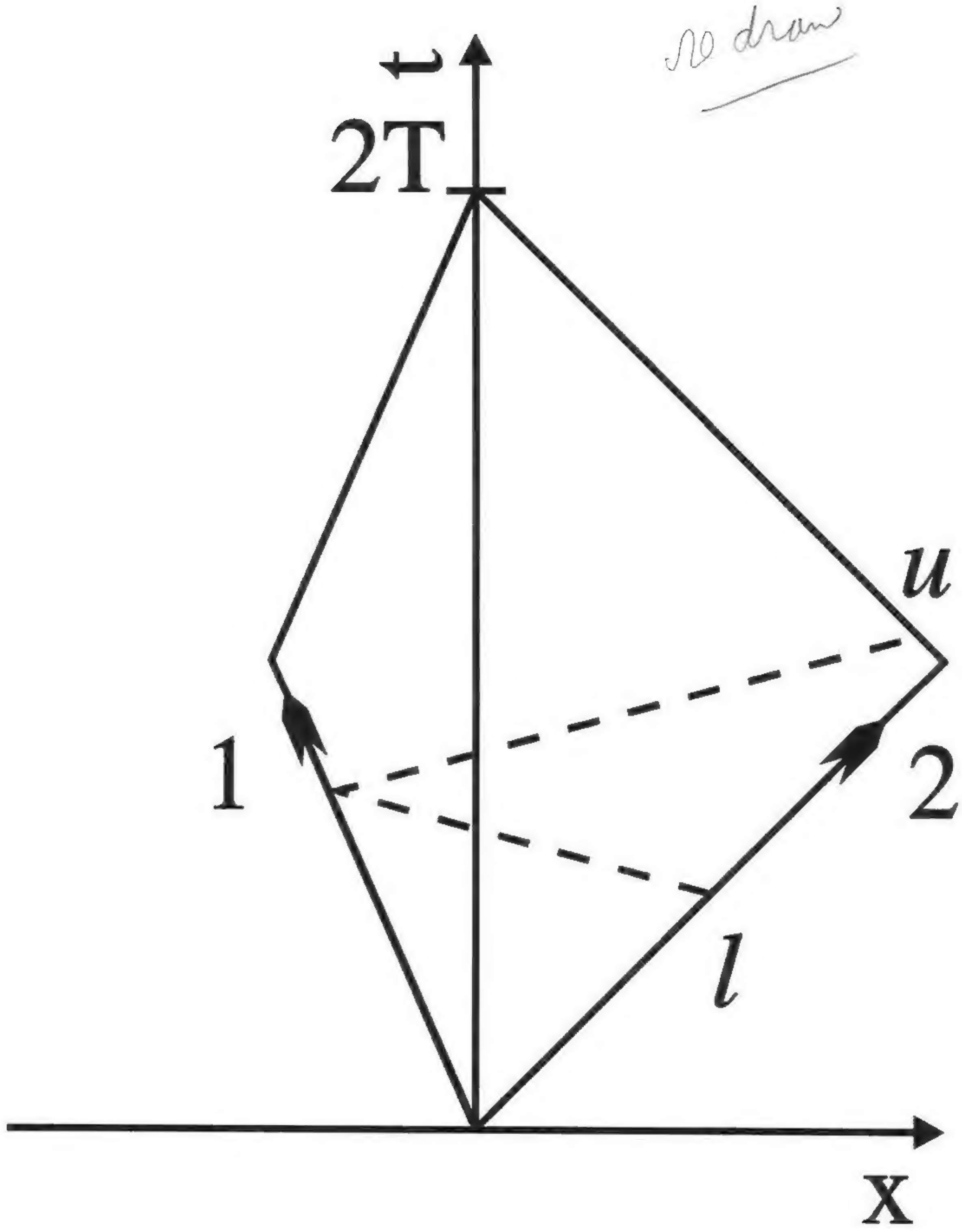

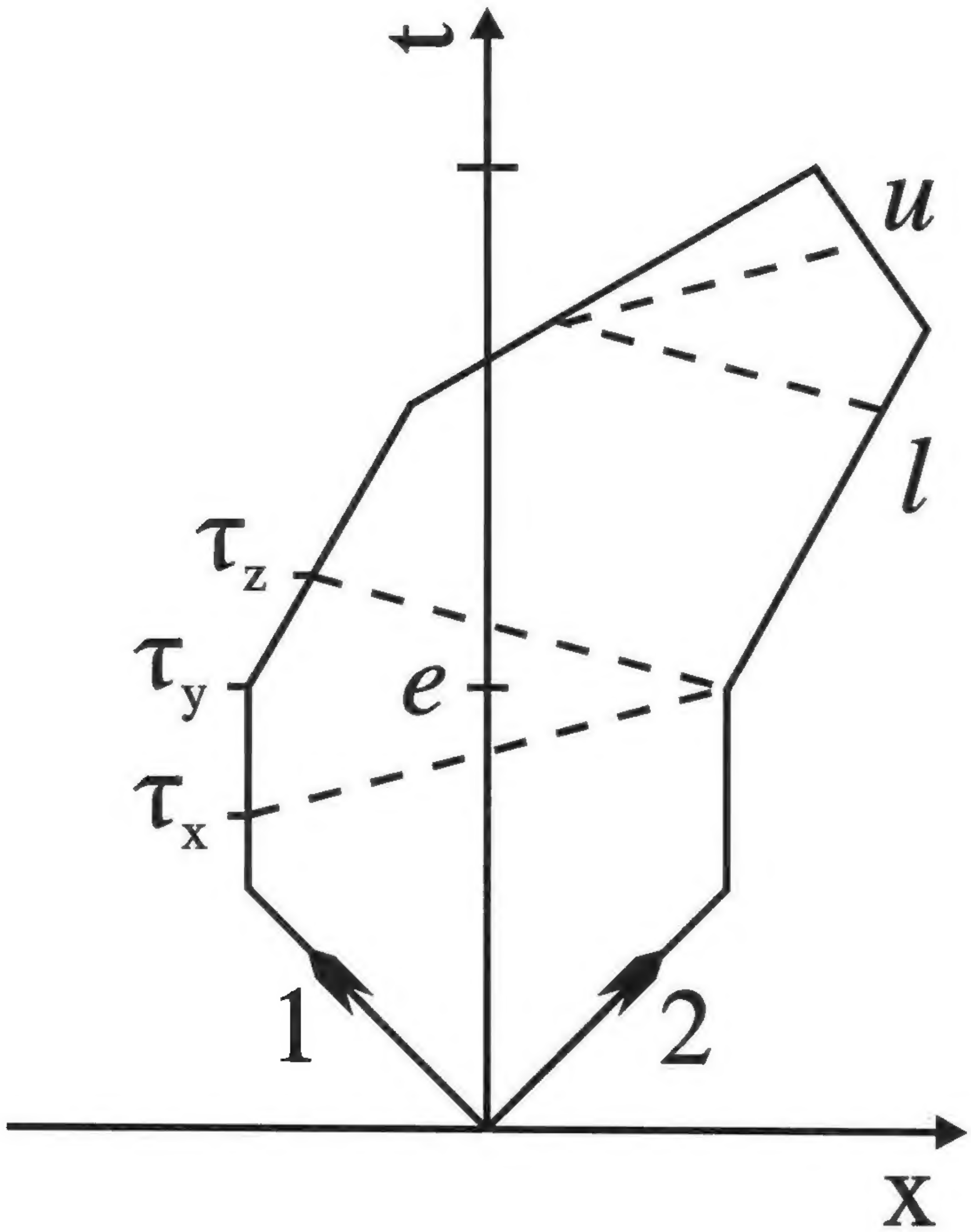
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